THE DEVELOPMENT OF A NEW MODEL FOR THREE-LEVEL SCM WITH CONSIDERATION OF PRICE-BASED RANDOM DEMAND AND THE EXISTENCE OF AT LEAST ONE COMPETITOR AND SOLVING IT WITH AN OPTIMIZATION APPROACH THROUGH AGENT-BASED SIMULATION

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Abstract. Considering the formation of the global economy in a competitive environment, as well as the requirement to produce diverse and timely products and meet customer demand, the need to design a supply chain with more flexibility and accountability is felt more than ever. In this research, the agent-based simulation technique is used to model the three-level supply chain. Then, considering a competitor in the model, the consumer expectation in each chain as a factor for the superiority of a chain compared to another chain was dealt with.

Keywords. Supply chain, baseline simulation, simulation optimization, competition between supply chains, globalization.

1. INTRODUCTION

The method and way of life in the current world has led to the emergence of a new word such as "globalization" that has opened up commodities to global markets. The most popular and common competitive factors in such markets are time and support. The national economy of many developed countries compared to developing countries includes small companies. Small and medium-sized enterprises (SMEs), due to their small size, face many problems, including low resources, limited ranges, and inadequate managerial experience. Therefore, such companies are not able to maintain themselves alone and therefore merge together to reduce the burden of problems.

Supply chain structures are one of the most common and effective structures for cooperation between small and medium size enterprises. A supply chain involves network of suppliers, manufacturers, distribution networks, and retailers, which are grouped together to control, manage, and develop the overall supply chain performance. At the highest level, a supply chain consists of two primary and integrated processes. 1. Production plan and inventory control process. 2. Distribution and logistics process. The supply chain has different risks and risks during design and operation. Some of these risks include:

- Uncertainty in supply and demand.
- Shorter life cycle of production time
- Increased use of partner in manufacturing, logistics and logistics services, which leads to a long and wide range of suppliers
- The need to understand risk in front of customers and suppliers
- Financial risk related to mismatch or excessive inventory, rework costs and penalties for non-delivery

To overcome such risks, some technological tools are used to enhance the supply chain performance. Among these tools, the most common ones are the supply chain management technique, which involves the planning and management of tools, information and financial flows in a network of manufacturers, distributors, and sellers and customers with the goal of reducing operating costs and increase services to customers.

To apply supply chain management techniques, critical supply chain performance measurement techniques must be carefully determined. Supply chain management involves the use of analysis tools such as system dynamics, optimization, and simulation. Theoretical models of supply chain behavior can be developed either by viewing data from the past or by collecting new data. Tests that study supply chain behavior are used to find out the causes of these effects on the testing of different scenarios. An alternative to performing experiments in the real system is to use a refinement of a system model for experiments. Many of the models mentioned in the literature review are the same. Today, the most effective and efficient instrument for measuring and determining the supply chain performance is simulation. Simulation can be applied to supply chain management, which is the concept of determining a process of a model. Like a project, a business, a mine, a forest or even one body organ to identify and understand system control factors or predict the future of system behavior. The purpose of the simulation is, in fact, to predict how the system evolves and reacts to its surrounding situations so that it can make any necessary changes that will help the system fulfill its requirements.

In recent decades, attention has been drawn to the use of simulation models in the supply chain. Different models of supply chains have been developed. In these studies, researchers have categorized supply chain management simulation into four different types.

- Spreadsheet simulation
- System dynamics (SD) simulation
- Discrete-event dynamic systems (DEDS) simulation
- Business games simulation

Spreadsheet analysis in general is a static-deterministic approach, so that it can be considered as a computing automation based on the supply chain data for analyzing SCS. Dynamic systems are tools that are specially designed for situations which are required to study the behavior change in the system at a comprehensive level [1].
Simulation of discrete dynamic systems is the modeling of a system in a situation in which the system variables immediately reflect the change over time. These points in time are the times when events occur, where events are defined as events that trigger the change in the state of the system [2, 3 and4]. Simulation game is a mixture of features of a competitive game, collaboration, attendees (participants) and rules. Obviously, these games are in a simulated environment. As these attributes confirm the goals in the simulation game, they are specifically developed for educational purposes [5].

In this research, it was attempted to design and analyze, as well as optimize, a three-level supply chain supply system, including supplier, factory, and distributor along with a competitor through agent-based simulation. It has also been attempted to design a framework based on the simulation of discrete events. Much of this design has been done by the agent-based simulation technique and reinforcement learning algorithm. In this regard, each supply chain agents learns how much to order or produce in any situation in terms of inventory position. Firstly, a general overview of the supply chain and various types of simulations has been addressed.

2. STATEMENT OF RESEARCH PROBLEM

Decisions in supply chain management include three types of operational, tactical, and strategic decisions that, for each of these types of decisions, well-known methods are used in terms of the type of decision. The agent-based simulation method used in this thesis is used in all three levels as an independent and efficient instrument. On the other hand, regardless of what level of decision we are considering, most modeling techniques that include math planning or simulation of system conditions examine systems in equilibrium. However, with the help of the agent-based simulation, systems that are in a state of imbalance or initial system conditions can be examined and analyzed. This unique property of this kind of simulation has made this type of simulation an efficient tool for analyzing bullwhip effect.

Another advantage of agent-based simulation is the inductive argument approach, which means that in this approach, the examination is begun with small levels of the system and then the entire system is dealt with. Since the supply chain system is a complex system that is subject to many factors (agents), it is difficult to understand its behavior, taking into account all relevant factors (agents). If, in the agent-based method, each component is independently defined in terms of behavioral and structural characteristics, which makes it much easier to check the system, which does not require a scenario, which would create conditions where behavior and actions of factors (agents) are far from the model-maker and scenario generator. Another advantage of this type of simulation is that the system can be considered dynamically, meaning that the system will evolve with the passage of time, in which case it will be possible to achieve optimal and robust responses. All the points mentioned above have made this kind of optimization method more widely used and attracted by many researchers and craftsmen.

Because of the advancement of technology and increased competition in global markets, the supply chain structure in the various industries is increasing, the application of simulation, and especially the agent-based simulation in academic circles, as well as among the most technologically advanced industries, is increasing. For example, in recent years, the multinational company Honeywell has attempted to use agent-based simulation for its global supply chain development and management. (Wagner et al., 2003). The company Boeing has also used the agent-based simulation to manage the supply chain of its own critical products (Brintrup et al., 2009). Another example of the application of the agent-based simulation in its supply chain is the successor project of General Electric Company in the field of pricing products in the supply chain (Brintrup, 2010).

The agent-based simulation can be considered as an effective tool for simulating supply chains with tensile structures. Because in such supply chains, due to the high cost of production and maintenance of products, and also the changing trend of the demand market, it is necessary to make an accurate estimate of the demand for the product at the appropriate time intervals and to provide the appropriate product accordingly. While the prediction of complex trends statistically is difficult and time consuming. In such a situation, agent-based simulation method with the help of artificial intelligence and data mining methods easily comes up with such issues. For example, the agent-based simulation in the optimizing and supply chain management of household electricity distribution networks (North et al., 2007), as well as oil supply chains, is used today (Sinha et al., 2011).

Typically, managers and decision makers about supply chain strategies should consider different
criteria in order to make decisions, which, on the other hand, is increasing with the more complex supply chains, and on the other hand, math and meta-heuristic algorithms are often not able to simultaneously optimize two target functions, or if this is possible, by increasing the number of target functions, the efficiency of these methods decreases. According to studies, cost and service standards are more important than the other criteria for active researchers in the supply chain. In addition to these two criteria, based on the principle of just in time (JIT) production, which was first used by Toyota, if the amount of inventory in the production and distribution centers decrease, the level of supply chain flexibility and vivacity will be increased. Of course, in the current situation where various industries are forced to buckle under varying conditions, in addition to finding the optimal solution, the sustainability (robust) of the answer is also important.

For such issues, using an optimization-based approach could be a suitable method. Because, firstly, using the simulation approach is an effective method for modeling issues with probability variables and uncertainty, and secondly, the simulation method is a flexible method, which can be easily combined with other methods and provides a powerful framework in the sustainable optimization. Similar approaches have been taken in the supply chain of Chinese automotive supply chain (Shi et al., 2012).

According to the stated content, the purpose of this research is to design and analyze as well as optimize the supply chain system of a three-level single product, including supplier, factory, and distributor, along with a competitor assuming the same factory in this study through agent-based simulation. An attempt has been made to design a framework based on the simulation of discrete events. Much of this design has been done by the agent-based simulation technique and reinforcement learning algorithm. In this regard, each supply chain agents learns how much to order or produce in any situation in terms of inventory position.

One of the known methods for modeling the behavior of the studied systems is using the simulation technique. Although the simulation technique, alone is not a tool for optimization (Nelson et al., 2004), however, its flexibility has led to other techniques such as meta-heuristic algorithms, artificial neural networks, response surface methodology, composition providing an efficient and valuable simulation-based optimization technique. Due to the advantages of simulation-based optimization, this method has been used in recent years to optimize and enhance the performance of complex systems, including military systems, aerospace and supply chain (Boesel et al., 2001).

There are different techniques and models for simulation. One of the most important of these models is the interaction model. Interaction, which is the main concept of this research and based on which the competitive image has been formed. The relationship (connectedness) and dependence between two or more independent entities is called interaction. Interaction in this dissertation is defined as the relationship between two or more supply chains at the given points, which simultaneously utilize the equipment, service or product provided by the service provider and in order to create more value for the final consumer by the used chain.

Because the performance of each of the components of the supply chain directly or indirectly affects other parts of the chain, so the interaction between the different sectors makes it possible to understand how the competition is created between the different chains.

Despite the ambiguity in the term value, the understanding of the term value is derived from the term Valuing. Definition of Valuing implicitly means the commitment of an organization to its customers in order to deliver the goods to a certain standard. The term standard is described as a combination of several products or services that are important to the customer, such as price, quality, convenience, and otherwise. However, in this thesis (dissertation), value has a more meaning. For example, for the purposes of accountability, it means the utility of the product or the service of each unit to the final consumer which means taking into account the cost of each unit for the value term. In other words, value means the economic relation between cost, convenience, quality and all promise of the manufacturer. However, this is due to value transmitted to customers. In fact, this competitive advantage is done to build and create competitive advantage throughout the supply chain. Therefore, the process of creating value along the supply chain created by this chain is necessary to achieve competitive advantage.

### 2.1 Research hypotheses
Two products A and B produced by two different and rival companies. The two products can be replaced each other.

The consumer population is 1,000 which do not take precedence over the two products.

Other consumers’ comments from and ads are important at 0.011.

Consumer views affect the choice of other consumers.

Consumers are contacting other consumers at a rate of 5 people a day.

Maximum waiting time in the absence of product is 2 days.

The retail capacity of has been considered 100.

Delivery time has been considered 2 days.

The decision variables are:

\[ x_{ij}^I: \text{The amount of inventory available in member i in period j} \]
\[ x_{ij}^B: \text{The amount of deficiency in the member i in period j} \]
\[ x_{ij}^O: \text{Order value issued in member i in period j} \]
\[ u_{ij}: \text{If demand for member i in period j is greater than available inventory, then value is 1 and otherwise zero.} \]
\[ y_{ij}: \text{If the member i issues the order in period j, the value is one and otherwise zero.} \]

3. RESEARCH THEORETICAL PRINCIPALS

Supply Chain is a collection of business processes and resources that transfer products and products from raw materials to end products, and eventually deliver products to the consumer. Supply chain management is defined as managing upstream and downstream activities with suppliers, distributors, and customers that give higher value to customers and less costs to them. The aim of supply chain management and the creation of a faster flow of higher quality providing relevant information that enabling suppliers to provide uninterrupted and timely of materials to customers

3.1. Performance criteria in supply chain management

Based on the issues outlined in the previous section, the criteria that can be used in the supply chain assessment are divided into six separate groups. These six groups are:

- Order planning criteria
- Suppliers Assessment Criteria
- Production Level Criteria
- Performance metrics for delivery
- Criteria for assessing customer satisfaction
- Supply chain cost metrics

This section reviews and examines the mentioned criteria.

3.1.1. Suppliers Assessment Criteria

The traditional performance criteria of suppliers over the years, based on the price of the products and services provided, the reject of the request and the delivery time, assess the performance of the suppliers. In recent years, traditional standards have been evolved into this area and in order to evaluate suppliers, more attention is paid to the criteria such as the reliability and quality of products and services. The new criteria in this field can be divided into three levels of operational, tactical and strategic.

The operational criteria include compliance with the scheduling, the ability to avoid complaints and the achievement of non-flawed delivery. Tactical criteria include the return on the time of the order cycle, the ordering process, the flow of liquidity, and the assurance of the quality and flexibility of the capacity. Strategic criteria include indicators such as standard procurement time, quality level and supplier pricing policy.

3.1.2. Production Level Criteria

- Production range: Usually a factory that produces a variety of products works to introduce a new product slower than a factory that produces finite products - factories that can promote the speed and reliability of product delivery.
- Utilization Capacity: Utilization capacity is crucial in determining the level of supply chain activities. The more utilization capacity, the more flexible production, and ultimately, meet the needs of customers increases.

- Efficiency of scheduling methods: Scheduling is based on the time, date and order of the work. This timing determines which source should flow to the system's operation, and which resource efficiency will have the greatest impact on supply chain performance. Since production schedules depend heavily on consumer demand and supplier performance, scheduling can have a significant impact on supply chain performance.

3.1.3. Performance metrics for delivery performance

- The total number of flawless bills: Each bill (invoice) represents the date, time, and delivery terms of the item to be received. Comparing previous bills with each other can indicate the correct delivery of the goods and otherwise identify the disruptions and try to resolve them.

- Flexible delivery system to meet customer specific needs: Flexible delivery and packaging have been agreed upon rules, and this could affect customers’ purchasing decisions and, thus, attract and retain customers.

- The total cost of distribution: the distribution sector is one of the most important components of the supply chain and the design of efficient distribution systems with the least possible cost is always the main concerns of the designers and supply chain researchers. It should be noted that the cost of the distribution network consists of different costs, the most important of which is the cost of transportation.

3.2.1. Customer Satisfaction Assessment Criteria

- Flexibility: The criteria by which different supply chains are compared with each other is a flexible criterion, and this criterion means the ability to provide a service that meets the needs of each customer.

- Customer response and answertime: The customer's response and answer time is how long the company responds to the customer's inquiry. For each customer, questions about the ordering situation and the potential problems of access to inventory or delivery are commonplace. For this reason, the accurate and prompt response to the needs of customers will lead to their satisfaction.

- The criteria of service delivery for the customer: The supply chain function does not end with the provision of goods for customers only, and customer service activities play an essential role in obtaining their satisfaction and getting feedback from them in order to improve the performance of the chain.

4. RESEARCH BACKGROUND

Wilson (2007) used a simulation approach to illustrate how transport influences supply chain performance and vendor managed inventory on two to five-level supply chain transport system using the. Vlachos et al. (2007) used this kind of simulation to analyze supply chain behavior in reverse (from retailer to factory). The proposed simulation model introduced an empirical tool that can be used in the evaluation of different, but consistent rules. Wang and Fong (2007) describe the promotion and impact of the real-life supply chain on industrial engineering students to help them gain experience in managing demand, and supply and stock in the supply chain.

Zhou et al. (2008) developed a two-level supply chain model based on game theory, combination of high-level factories and retailers as a low level in the supply chain. It was assumed that the supply chain expressed in fuzzy space works closely with customer demand and producer costs. To do this, they considered two different game structures: the manufacturer and the retailer collaborate and are known as a single company, and the producer acts as a dominant player in the supply chain such as the Stackelberg’s leader.

Sun et al. (2012) in their research used a different approach to agent-based simulation to reduce the supply chain financial risks. In the study, in addition to reducing financial risks, efforts have been made to provide solutions to reduce the transmission of financial fracture among supply chain members. Li and colleagues (2010) emphasized the issue of supply chain marketing and pricing, and also in their research the agent-based simulation method to show that, in the case of fall in prices, not only the former profit can be maintained, but also the profit is sustained. Chan et al., 2010 have used agent-based simulation to review strategic decisions regarding the adaptability or flexibility of the supply chain, and in this regard, the criterion of service level and productivity has been taken into account in
measuring various strategies. Sinha and colleagues modeled the oil supply chain as agent-based system and optimized resource allocation in order to reduce inventory.

In an article entitled "uncertainty chain management in the context of explaining supply chain competition, Fang and shou (2015) studied the competition between two supply chains in uncertain conditions. In this research, each chain consists of a supplier and an exclusive retailer in conditions of uncertainty. They showed how the grading of different levels of supply and the intensity of competition affected by the decision-making equilibrium order of how to submit the contracts, as well as the determination and selection of the concentration of the chain.

Lia and Lee (2016), in a paper titled chain-to-chain competition on product sustainability, studied the game model of twosustainable supply chainsin competition in product sustainability, system equilibrium structure of two chains, as well as the extraction and production of managerial knowledge. They analyzed the supplier and constructor's time in the competitive reverse supply chain, degree of sustainability, desires and profits in the three structure of the two-chain system. They analyzed the supplier and constructor's time in the competitive reverse supply chain, degree of sustainability, desires and profits in the three structure of the two-chain system.

Hung et al. (2016), in a paper entitled equilibrium analysis of pricing competition and cooperation in the supply chain with a joint manufacturer and monopoly of retailers, examined pricing-based competition and issues of collaboration in a two-level supply chain with a joint manufacturer and exclusive retailers. Because of this, they built six decentralized game models to examine how pricing strategies (Bertrand and collusion) and power structures (dominant manufacturers, dominant and non-dominant retailers) affect the performance of supply chain members.

5. FINDINGS OF THE RESEARCH

5.1. Modeling of product demand

Since the demand for the product in question in both manufacturers A and B has a great dynamic and over time, its amount and dispersion increase, in order to model the multi-resolution method is used. Also, to determine the policy of a fixed point of sale, a fixed review period and ordering based on demand forecast has been used. In the design of this model, a full design has been used to create a doable space. Also, multi-resolution method has been used to model demand for products. This method, which is known as a non-parametric method, is presented by Kuhel et al., 2001 to model non-homogeneous Poisson processes with periodic changes and long-term trends. Since this method is known as a non-parametric method, it is independent of statistical parameters and can easily be used in a variety of issues. In addition, the multi-resolution method can be used to model processes that have several periodic variation patterns. Another advantage of this method is its ability to model non-symmetric patterns. In addition to the many features that make this unique, its performance is also very simple. In order to use this, we first need to divide the intervals that include periodic changes. For example, if we have a multi-year course (period), we can divide it into one-year courses, and divide the system of each one-year course into twelve one-year periods, and each one year period to four courses of one month and also divide each one month period into four weekly periods, then we obtain the cumulative function of the studied variable for each of these periods to the extent that the range of functions obtained is in the interval between zero and one. Then, by combining the functions obtained nesting mode, you can regenerate the studied variable. In the present study, the operation was performed as a model for the single-product demand for the five-year period. The five-year course (period) is divided into one-year courses, as well as one-year courses. Since for the intended product a period of five years can be extracted each of which has the same monthly pattern, the monthly average monthly demand estimates are used to estimate the monthly average, as well as, in order to more accurately estimate a weight is assigned to each month which is equivalent to the variance of the values obtained for five consecutive years. In this way, in the estimation process, for month in which given demand for them is less dispersed will be more important. The process of demand estimating and modeling is done by the curved tool box of the R2012a software, MATLAB.

- Estimation and Modeling of Demand Model of Product A

The cumulative demand function for product A is as follows, which is a polynomial function of degree three.
Figure 1: Cumulative Demand Function of Annual Demand of Product A

Also, the value of $R^2$ for the fitted function is 1.

Linear model Poly3: $f(t) = p_1 t^3 + p_2 t^2 + p_3 t + p_4$

Coefficients (with 95% confidence bounds):

\[
p_1 = 4.85e-23 \ (4.158e-23, 5.542e-23)
\]
\[
p_2 = -6.236e-19 \ (-7.005e-19, -5.467e-19)
\]
\[
p_3 = 1 \ (1, 1)
\]
\[
p_4 = 1.961e-13 \ (1.059e-13, 2.864e-13)
\]

Goodness of fit:

- SSE: 5.717e-22
- R-square: 1
- Adjusted R-square: 1
- RMSE: 8.862e-13

Estimation and modeling of demand pattern of product B

The cumulative demand function for product A is as follows, which is a polynomial function of degree three. Also, the value of $R^2$ for the fitted function is 1.

Figure 2: Cumulative Demand Function of Annual Demand of Product B
Linear model Poly3:

\[ f(t) = p_1 t^3 + p_2 t^2 + p_3 t + p_4 \]

Coefficients (with 95% confidence bounds):

\[
\begin{align*}
p_1 &= 1.285e-22 (7.608e-23, 1.81e-22) \\
p_2 &= -1.503e-19 (-4.436e-19, 1.431e-19) \\
p_3 &= 1 (1, 1) \\
p_4 &= -1.123e-13 (-1.978e-13, -2.684e-14)
\end{align*}
\]

Goodness of fit:

- SSE: 5.037e-22
- R-square: 1
- Adjusted R-square: 1
- RMSE: 8.318e-13

Given that \( f(t_m) \) is the cumulative monthly demand function and \( f(t_y) \) is the annual demand function, if we show the demand for month \( t_m \) from year \( t_y \) with \( Demand(t_y, t_m) \), then, based on the quasi-code shown in Fig. 3, we can produce demand quantities for both of these factories.

\[
For \ t_y = 1 \ to \ 5 \\
\quad \text{Demand of year } (t_y) = (f(t_y) - f(t_y - 1))^* (\text{total demand})
\]

\[
For \ t_m = 1 \ to \ 12 \\
\quad \text{Demand of } (t_y, t_m) = (f(t_m) - f(t_m - 1))^* (\text{Demand of year } (t_y))
\]

In order to validate the methodology used in modeling of the demand for the product, Figure 4 compares the monthly production demand of the factory B with the quantities generated by the multi-resolution method and, as can be seen, the actual values are well matched to the production values.
5.2. Lead time for commodity (product) renewal

When the lead time for commodity (product) renewal rises, the amount of confidence commodity (product) required by the buyer to be kept is also increased by squared squares. The lead time performance of suppliers can directly affect the amount of confidence commodity (product) required by relationship (equation) 1.

Desired cycle service level (CSL)

Mean demand during lead time $D_L$

Standard deviation of demand during lead time $\sigma_L$

Safety stock (SS)

(Equation 1)

$$ROP = D_L + ss$$

$$ROP^3 = D_L + ssst = F^{-1}_s(CSL) \cdot \sigma_L$$

5.3. Timely performance

The timely performance influences the variability or the lead time variability. A trusted supplier has low lead time changes and an unreliable supplier with a high lead time. When the lead time changes are increased, the confidence level is rising rapidly (equation 1). Timely performance can be transformed into the lead time variability (Equation 2).

Mean demand in each period $D$:

Standard deviation of demand in each period $\sigma_D$

Mean lead time for commodity (product) renewal $L$:

Standard deviation of lead time for commodity (product) renewal $s_L$:

$$D_L = D_L \sigma_L = \sqrt{L \sigma_D^2 + D^2 s_L^2}$$

5.4. Mathematical analysis of lead time

As previously explained, each supply chain has different and various stages. For this, there are 3 basic and fundamental stages that include raw materials (people ($S_0$)) manufacture line (product A

1. Mean demand during lead time
2. Standard deviation of demand during lead time
3. Reorder Point
or B) and warehouse (exit from system (S_1)). It is clear that before each of these stages and then those ranks are created because the lead time in the queue is meaningful. For better visibility, figure 5 is considered as the schematic view with the queues created for this problem. (T Means transport)

\[ \lambda_1 = \lambda^0 q_1 \quad \lambda_2 = \lambda^0 q_2 \quad q_1 + q_2 = p_1 + p_2 = p_3 + p_4 = 1 \]

**Figure 5 Schematic representation of the problem model with formed queues**

For the formed queues, the M/M/1 model is considered as the basic model, which is further described and defined. The M/M/1 queue model has a Poisson acquisition process for customer input and an exponential distribution for service delivery. The description of the situation for this model is simple, so that the time between the two consecutive exponential inputs is and the service time is exponential and there is only one consumer in the system. Because people as source, whether they want product A or B, they are shown in this section with only one source. And then go into two queues of commodity (product) A or commodity (product) B. After logging in, there are three stages (There are three stages (steps) in the simulation process (Figure 6).

**Figure 6 Problem Model in Simulation Software**

The first stage is entry into the factory and then the warehouse of the manufacturer of commodities (products) A or B. Therefore, there are two inputs to the system (A or B), which are queuing in the network that are being asked by the consumer. This request waits to be answered. The quantity of delivered goods (commodities) in each delivery is assumed to follow a uniform (same) distribution.
The rate of entry (input) of two types of consumer is \( \lambda_1, \lambda_2 \). And the entry (input) rate of \( S_0 \) is also considered as \( \lambda^{(0)} \). \( S_0 \) and \( S_1 \) are hypothetical and are used only for entry (input) and exit counting. The probability of entering into \( Q_0 \) and \( Q_5 \), \( q_1 \), and \( q_2 \) is considered. Because the sum of probabilities in each system is considered equal to one, it is clear that this system also adheres to this rule, namely \( q_1 + q_2 = 1, \lambda_1 = \lambda^{(0)} q_1 \) which represents the rate of entry (input) to \( Q_0 \) and so \( \lambda_2 = \lambda^{(0)} q_2 \) entry (input) rate to \( Q_5 \). Service rate of \( A_4 \) and \( A_5 \) are called \( \mu_4 \) and \( \mu_5 \). After receiving service from \( A_4 \), they will be in \( Q_4 \) and \( Q_5 \) queues with \( P_4 \) and \( P_5 \) probabilities. Obviously, \( P_1 + P_2 = 1 \), so the entry rate to \( Q_0 \) will be \( \lambda_1 P_1 \), and the entry rate is \( Q_0 \). After receiving service by \( A_5 \), the applicant will enter \( Q_1 \) at the entry rate \( \lambda_2 P_2 \). The service rate is considered in \( A_6 \) and \( A_7 \), \( \mu_2 \), and \( \mu_5 \). Because \( Q_5 \) and \( Q_2 \) queues are serially connected, the request served by \( A_5 \) will be served by \( A_7 \) after waiting in \( Q_1 \) queue. The service rate to \( A_6 \) is \( \lambda_2 P_2 \). After taking service by \( A_7 \), item \( A \) enters at the same rate \( \lambda_2 P_2 \) to \( Q_1 \) queue and is served by server \( A_5 \). Service rate on server \( A_1 \) is \( \mu_5 \). After receiving service by server \( A_5 \), the product \( A \) enters \( Q_17 \) and \( Q_4 \) queues with \( P_3 \) and \( P_4 \) probabilities. Obviously, in this section, \( P_1 + P_2 = 1 \). The entry rate in \( Q_17 \) will be \( \lambda_2 P_4 \) and also will be \( \lambda_2 P_4 \) in \( Q_18 \).

Because \( Q_9 \) and \( Q_17 \) are series, so product \( A \) will be served by server \( A_9 \), and after serving by \( A_17 \) server, and after waiting in \( Q_6 \) queue. Serving rates on servers \( A_{17} \) and \( A_9 \) are considered to be \( \mu_2 \) and \( \mu_4 \). The entry rate to \( Q_6 \) is \( \lambda_2 P_3 \). After receiving the service by server \( A_9 \), the commodity (product) \( A \) enters queue \( Q_12 \) with the same entry (input) rate \( \lambda_2 P_3 \) and by server \( A_1 \), whose service rate is \( \mu_6 \). Similarly, \( Q_8 \) and \( Q_{10} \) queues will also be the same. Therefore, the product \( A \) which served by the \( A_{18} \), server, will also be served by the server \( A_{10} \), after waiting in the \( Q_{10} \) queue. Serving rates on servers \( A_{18} \) and \( A_{10} \) are equal to \( \mu_4 \) and \( \mu_2 \). The entry rate in \( Q_{10} \) will be \( \lambda_2 P_4 \). After receiving the service by the \( A_{10} \) server, item (product) \( A \) entered at the same rate \( \lambda_2 P_4 \) into \( Q_{14} \) queue and served by server \( A_{14} \) whose service rate is \( \mu_4 \). At the end, the product \( A \) is given after service by servers \( A_{11}, A_{12}, A_{13} \), and \( A_{14} \), to the applicant for the product \( A \) at a rate of \( \lambda^{(0)} \). Based on Fig. 6, the following results are obtained.

(Equation 3)

\[
\lambda_2 = \lambda^{(0)} q_1, \ 
\lambda_2 = \lambda^{(0)} q_2
\]

\[
\lambda_1 + \lambda_2 = \lambda^{(0)} p_1 + p_2 = p_3 + p_4 = q_1 + q_2 = 1
\]

Now it is assumed for greater control that \( \mu_1 = \mu_1 \) and \( \mu_2 = \mu_3 = \mu_4 = \mu_5 \).

A node is defined by its queue and server.

The nodes in step \( I \) \( (Q_4, A_4), (Q_5, A_5) \)

The nodes in step (stage) \( II \) are:

\( (Q_6, A_6), (Q_{17}, A_{17}), (Q_9, A_9), (Q_{15}, A_{15}), (Q_7, A_7), (Q_{18}, A_{18}), (Q_{10}, A_{10}) \)

The nodes in stage \( III \) are \( (Q_{11}, A_{11}), (Q_{12}, A_{12}), (Q_{13}, A_{13}), (Q_{14}, A_{14}) \). Each activity belongs to one of the processes in the relevant stage. Activities of \( A_{17}, A_{15}, A_{18}, A_{11}, A_{13}, A_{14}, A_{10} \) are related to the transport activity. From the perspective of consumers, the supply chain is equivalent to a queue. A queue of arrived requests-requests that are waiting to be answered. Requests are categorized by events, numbers, and delays.

Because events (\( \lambda \)) are a statistical phenomenon in nature with the distribution of Poisson, and the number of tasks of any event that needs to be done on it, uses a uniform distribution in nature, and delays, which are the most important quality identifier, are considered as the random variable \( Z \) that the function has both \( X \) and \( Y \).

\( X = \) random variable of event occurrence of each request.

\( Y = \) Random Variable of the number of requested goods.

\( Z = XY \), which represents the event time with the number of items (goods) requested.

The cumulative distribution of \( Z \) is:

\[
F_Z(Z) = P(Z \leq Z) = \int_{A_2} \int_{x} f_{XY}(X, Y) \, dx \, dy
\]

Which \( A_2 \) is a subset of \( R^2 \), which is \( A_2 = \{(X, Y) \mid F(X, Y) \leq Z \} \). Since \( f_X(x) \) and \( f_Y(y) \) are independent, so \( f_X(x) f_Y(y) \). So \( F_Z(Z) = \int_{A_2} \int_{x} f_X(x) f_Y(y) \, dx \, dy \).

If \( X \) is considered as a random variable with an exponential distribution, and let \( Y \) be a random variable with uniform distribution, we will have a uniform distribution between \( a \) and \( b \) (\( b < a \)).
\[ f_x(x) = \begin{cases} ye^{-\lambda x}; & x \geq 0 \\ 0; & x < 0 \end{cases} \]
\[ f_t(y) = \begin{cases} 1 & a < y < b \\ 0; & \text{otherwise} \end{cases} \]

The zone \( D_z \) such that \( z < xy < z + dz \) is the part of the curve between the outer boundary of two rectangular points \( y = \frac{z}{x}, x = \frac{z+dz}{x} \). The coordinates of a point in this zone are \( \frac{z}{x}, x \) and \( \frac{dz}{dy} = \frac{1}{|x|} \). So \( \frac{dy}{dx} = \frac{1}{|x|} \). Since two variables of \( X \) and \( Y \) are independent, the probability of the density function \( Z \) will be equal to:

(Equation 4) \[ f_z(z) = \frac{\lambda}{b-a} (E_1 \left( \frac{\lambda z}{b} \right) - E_1 \left( \frac{\lambda z}{a} \right)) \]

With the change in the variable in the fraction of the equation 4 and change \( t = \lambda x \) and \( dt = \lambda dx \) we will have:

(Equation 5) \[ f_z(z) = \frac{\lambda}{b-a} \left( E_1 \left( \frac{\lambda z}{b} \right) - E_1 \left( \frac{\lambda z}{a} \right) \right) \]

Where \( E_1(x) = \int_x^\infty e^{-u} \frac{du}{u} \) is defined exponential fraction by \( E_n(x) = \int_1^\infty e^{-zt} dt \) in \( n = 1 \) [72]. Now \( E_1(x) = -E_1(-x) \) when \( E_1(x) = - \int_x^\infty e^{-t} dt \) is the operator exponential fraction. By changing \( E_1(x) \) and \( E_i(x) \) in the equation 5 we will have:

\[ f_z(z) = \frac{\lambda}{b-a} \left( E_1 \left( \frac{\lambda z}{b} \right) - E_1 \left( \frac{\lambda z}{a} \right) \right) \]

The mean event time \( E(Z) \) will be equal to:

(Equation 6) \[ E(Z) = \int_0^\infty z f_z(z) dz = \frac{\lambda}{b-a} E \left( \frac{\lambda z}{b} \right) \]

The integral of equation 6 will be:

\[ E(Z) = \frac{\lambda}{b-a} \left[ \frac{\lambda^2}{2} E \left( \frac{\lambda z}{a} \right) - E \left( \frac{\lambda z}{b} \right) \right] + \frac{az}{2\lambda} + \frac{a^2}{2\lambda^2} \] \[ \exp \left( -\frac{\lambda x}{a} \right) - \frac{b^2}{2\lambda^2} \exp \left( -\frac{\lambda x}{b} \right) \] \[ 0 \]

(Equation 7)

Therefore, the arrival time of the requests (occurrence and number) the distribution will have an exponential with the mean \( E(Z) \). The service life of the requests is non-dependent and distributed equally to random variables; the usual (typical) distribution for requests is a view with a mean \( \frac{a^2}{\mu} \), where \( \mu \) is the service rate.

The service system is assumed to follow the FCFS\(^4\) method. \( N(t) \) is considered to be the number of requests that constitute the queue at time \( t \). Then \( \{N(t) | t \geq 0\} \) is a birth-death process with a minimum input process:

(Equation 8) \[ A_k = \lambda(0) = \frac{1}{E(Z)} = \frac{2\lambda}{b+a} \]

And the service rate \( \mu; k = \mu; K \geq 1 \). The ratio is also equal to

\[ \rho = \frac{\text{Serving mean time}}{\text{Mean time between inputs}} = \frac{A_k}{\mu_k} = \frac{1}{\mu E(z)} = \frac{2\lambda}{\mu(b+a)} \forall a, b > 0, b > \alpha \]

(Equation 9)

The value of \( \rho \) is an important parameter called the traffic intensity and is usually expressed by the Erlang distribution. From the birth-death process for the homogeneous time series of Markov chains, the uniform probability of having \( k \) order in a system with continuous entries

\[ P_k = \left( \exp \left( -(1-\rho) \right) \right)^k \frac{\rho^k \exp(-k(1-\rho))}{\rho^k} \forall a, b > 0, b > a \]

(Equation 10)

The sum of the equations 10 from 0 to \( \infty \) is equal to 1. \( P_0 = \exp(1-\rho) - 1 \) resulted in \( \rho < 1 \). When the traffic intensity is less than unit, the level of service engagement will be equal to

\[ U_0 = 1 - P_0 = 2 - \exp(1-\rho) \]

\(^4\)First Come First Served
This equation can indicate the mean and variance of the number of service recipients in the system as follows:

\[
E[N] = \sum_{k=0}^{\infty} k \Pi_k = \Pi_0 \sum_{k=0}^{\infty} k \exp(-k(1 - \rho)) = \frac{1}{1 - \exp(-(1 - \rho))} \quad (Equation \ 11)
\]

And

\[
\sigma_N^2 = \sum_{k=0}^{\infty} (k - E[N])^2 \Pi_k = \frac{\exp(-1 - \rho))}{(1 - \exp(-(1 - \rho))}^2 \quad (Equation \ 12)
\]

If the random variable \( R \) is the response time (i.e., the time the customer enters to the time of leave). In order to calculate the mean service time \( E[R] \) of the case, which states that the mean number of recipients in the same conditions in the queue is equal to the product of the entry rate and the mean response time, so

\[
E[R] = \frac{E[N]}{\lambda} = \frac{1}{\lambda(1 - \exp(-(1 - \rho)))} \quad (Equation \ 13)
\]

Now if \( W \) is considered as the lead time in the queue. The mean lead time will be equal to:

\[
E[W] = E[R] - \frac{1}{\mu} = \mu - \lambda + \frac{\lambda \exp\left(-\left(1 - \frac{2\lambda}{\mu(b + a)}\right)\right)}{\lambda \mu \left(1 - \exp\left(-\left(1 - \frac{2\lambda}{\mu(b + a)}\right)\right)\right)} \quad (Equation \ 14)
\]

5.5. Statistical analysis of supply chain performance

In order to more and more investigate the probable behaviors of supply chains in this research according to 50 samples taken from the total cost of the 5-year horizon (perspective) of the supply chain and the customers’ mean lead time to likelihood fit for the total cost of supply chain and the customers’ mean lead time has been addressed. The results of fitting probability function for supply chain demand for the two products A and B for these two products are presented in Table 1 and Table 2, respectively. Also, the fitness of customer mean time for both products is also presented in Tables 3 and 4.

Table 1 Product A demand

<table>
<thead>
<tr>
<th>Functional</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guasson</td>
<td>0.001095</td>
</tr>
<tr>
<td>Polynomial</td>
<td>-0.0005453</td>
</tr>
<tr>
<td>Power</td>
<td>-0.0002681</td>
</tr>
<tr>
<td>Rational</td>
<td>-0.0518</td>
</tr>
<tr>
<td>Smoothing Spile</td>
<td>0.9884</td>
</tr>
<tr>
<td>Sum of Sine</td>
<td>-0.001097</td>
</tr>
<tr>
<td>Weibull</td>
<td>-0.7891</td>
</tr>
</tbody>
</table>
Table 2 Product B demand

<table>
<thead>
<tr>
<th>Functional</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guasson</td>
<td>0.001676</td>
</tr>
<tr>
<td>Polynomial</td>
<td>0.001638</td>
</tr>
<tr>
<td>Power</td>
<td>0.004322</td>
</tr>
<tr>
<td>Rational</td>
<td>0.001815</td>
</tr>
<tr>
<td>Smoothing Spile</td>
<td>0.985</td>
</tr>
<tr>
<td>Sum of Sine</td>
<td>0.00149</td>
</tr>
<tr>
<td>Weibull</td>
<td>0.6564</td>
</tr>
</tbody>
</table>

Table 3 The fitness results of distribution function of the customer’s mean lead time for products A and B.

<table>
<thead>
<tr>
<th>Functional</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guasson</td>
<td>0.4269</td>
</tr>
<tr>
<td>Polynomial</td>
<td>0.788</td>
</tr>
<tr>
<td>Power</td>
<td>-------</td>
</tr>
<tr>
<td>Rational</td>
<td>0.4293</td>
</tr>
<tr>
<td>Smoothing Spile</td>
<td>0.8392</td>
</tr>
<tr>
<td>Sum of Sine</td>
<td>0.788</td>
</tr>
<tr>
<td>Weibull</td>
<td>-------</td>
</tr>
</tbody>
</table>

As shown in the tables above, the Guasson distribution function is the best choice for both products. The parameters of this distribution for the mean lead time for product A are $\mu = -21.0650$ and $\sigma = 622.0088$ and for product B is $\mu = -7.7700$ and $\sigma = 139.6427$. If we consider the mean and regression variables for both products, we can show the distribution functions of these two products by equations (4.1) and (4.2) and also plot the corresponding graphs in Figures 7 and 8.

$$f(x_1, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_1 - \mu)^2}{2\sigma^2}}$$
\[ f(x_1, -21.650, 622.0088) = \frac{1}{622.0088\sqrt{2\pi}} e^{-\frac{(x_1 + 21.650)^2}{2(622.0088)^2}} \] (4.1)

\[ f(x_2, -7.7700, 139.6427) = \frac{1}{139.6427\sqrt{2\pi}} e^{-\frac{(x_2 + 7.7700)^2}{2(139.6427)^2}} \] (4.2)

Figure 7 The product density distribution function of the product A

Figure 8 The product density distribution function of the product A

6. SUMMARY AND CONCLUSION

Today, supply chain issues are one of the topics of the day in the field of industrial and services.
Because each supply chain involves a complex network of different levels, including suppliers, manufacturers, and distributors, and has led to complex issues in this area including the competition issues among independent supply chains and how to test this competition. This is one of the most important issues in this field and has therefore doubled in importance. However, due to the complexity of this topic, little has been done in this regard.

Given the increasing diversity of products and the multiplicity of supply chains, as well as the competitiveness of commercial markets, the dynamics of access to supply chain systems with superior features are more evident, hence, this research using the master’s thesis and the doctoral thesis in Iran for the first time has applied the agent-based simulation of a three-level inventory system with competitor. In the research, the problem of distribution function in the chain has been well seen. In addition to the efforts made in this study to apply the agent-based simulation in order to optimize the supply chain system, a solution is also needed to obtain the consumer’s lead time in the supply chain and considering this agent as to examine the competition between independent supply chains has also been used. Given the fact that the topic of agent-based simulation is a new and emerging topic, and the current research is the first official post-graduate study, there are many relevant issues for future research. Among the potential of the issues that are considered by the researcher are:

- Development of the combined model of the agent-based and dynamic system in order to complete the proposed model.
- Using a reinforcement learning algorithm to achieve pricing systems in the supply chain.
- Using agent-based simulations in the management of technology and supply chain of modern knowledge.

REFERENCES


